# 

# Explainable Trajectory Representation Based On Dictionary Learning

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#### Background.

- Trajectory representation learning provides great opportunities to understand vehicular traffic patterns.
- Downstream tasks includes Trajectory compression, Trip time estimation, Public transportation route planning, etc[1].



Transforming a trajectory into an embedding vector

#### Motivation.

- Embedding generated by deep-learning method is usually a dense vector whose dimension lacks semantic information.
- It is difficult to interpret the learned representation and use in applications[2].

#### Algorithm

- 1. Relax the binary constraint and get the fractional solution  $R^*$
- 2. Obtain the rounded solution  $R^r$  as follows:
- Algorithm 1 randomized rounding
- **Input:** *M*: trajectory matrix; *D*: pathlet matrix;  $R_0$ : initial solution;  $\epsilon, \theta$ : hyper parameters;

**Output:** Optimal binary matrix  $R^r$ 

- 1: # Step1, we compute the fractional solution  $R^*$  using gradient descend.
- 2: initial  $R_0 = 0$ ;

#### 3: repeat

- 4: compute gradient directions  $g_k = \nabla f(R_k)$ ;
- 5: update the decision matrix  $R_{k+1} = R_k \alpha g_k$ ;
- 6: clip the result to make sure  $0 \le R_k \le 1$ ;
- 7: **until**  $(|f(R_k) f(R_{k-1})| < \epsilon)$
- 8: #Step2, we compute rounded solution  $R^r$  based on  $R^*$ .
- 9:  $P(R_{i,j}^r = 1) = min(1, \theta R_{i,j}^*)$

**Probability bound.** Given the size of dataset |T|, trajectory matrix M, pathlet matrix D and trade off parameter  $\lambda$ . Then for constant parameter  $\theta$ , we have the following bound on the cost of  $R^r$ :

$$P[C(B^r) < 2\theta \frac{\lambda + 1}{2} C(B^*) \text{ and } DB^* > M] > \frac{1}{2} - |T|e^{-\theta}$$

#### Purpose.

- Extract common trajectory segments(called pathlet[3]) as a dictionary.
- Represent trajectory by concatenating pathlets from this dictionary.
- Generate semantic trajectory representation vectors, each dimension corresponding to a mobility pattern.



#### Evaluation.

- This dictionary should be able to reconstruct all trajectories.
- Smaller dictionary is better, which means less redundant information.
- Average number of pathlets used to reconstruct trajectory should be as small as possible.

### **Problem Formulation**

**Terminology.** Given a trajectory dataset T on roadmap  $G = \langle E, V \rangle$ , P(t) is used to describe all possible subpath p of  $t \in T$  and  $\overline{P} = \bigcup_{t \in T} P(t)$  refers to the whole pathlet space. The task is to find best pathlet dictionary  $P \subset \overline{P}$  according to evaluation indicators.



#### $P[C(R^r) \leq 2\theta - \frac{\lambda}{\lambda} C(R^r) \text{ and } DK \geq M] \leq \frac{1}{2} - |I|^{\epsilon}$

#### Experiment & Result

#### Case study.

- Pathlets are visualized to verify if common patterns are found.
- Pathlets in Fig(a) refers to turning around or turning left on the overpass, which is consistent with our cognition in life.
- Fig(b) illustrates how a trajectory is reconstructed by pathlets.



(a)Common mobility pattern on an overpass

(b)Trajectory decomposition using pathlets

#### Effect of $\lambda$ .

- The average number of pathlets need to construct trajectory decreases as  $\lambda$  increases, which means longer pathlet are selected.
- The algorithm prefers a more compact dictionary with a smaller  $\lambda.$



![](_page_0_Figure_55.jpeg)

Matrix are used to formulate the objective and constraint

- R is decision matrix, corresponding to dictionary P.
- Matrix D and M record the cover relationship between  $T, E, \overline{P}$

Based on this, the problem can be formulated as follows:

![](_page_0_Figure_60.jpeg)

#### Comparison.

• Proposed method outperforms heuristic method and dynamic programming method[3] under different lambda on synthesis dataset .

#### Conclusion

- A novel dictionary learning based method with theoretical probability bound analysis is proposed to solve the trajectory representation problem.
- Explainable trajectory representation are generated, providing a deeper insight into mobility patterns.
- This method will be applied in more real-world applications to verify its generality in future work.

#### Reference

[1] T.-Y. Fu and W.-C. Lee, "Trembr: Exploring road networks for trajectory representation learning," in ACM Transactions on Intelligent Systems and Technology, vol. 11, pp. 1–25.
[2] P. Y. e. al, "T3s: Effective representation learning for trajectory similarity computation," in 2021 IEEE 37<sup>th</sup> International Conference on Data Engineering (ICDE). IEEE, 2021.
[3] C. Chen . e. al, "Pathlet Learning for Compressing and Planning Trajectories," in Proceedings of the 21st ACM SIGSPATIAL.

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