Low Resolution Quantization for Wireless Communications

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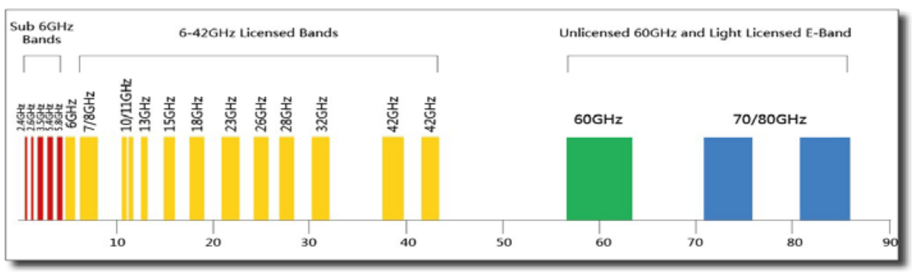
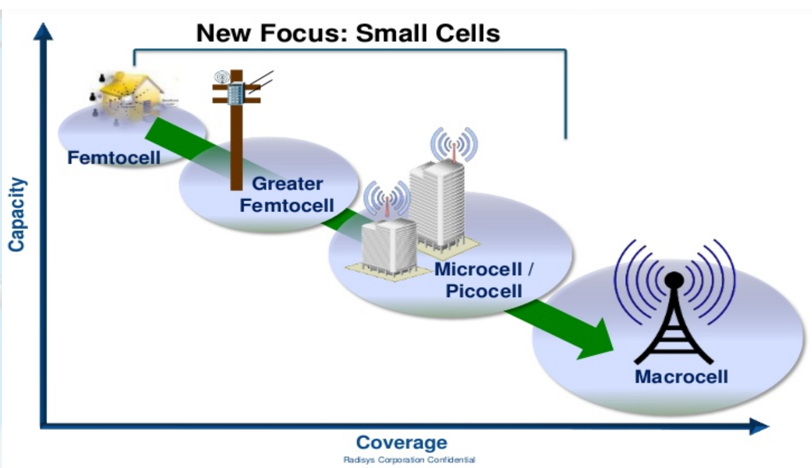
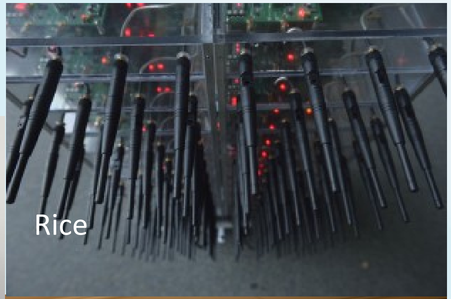
The Road to Gigabit Wireless (5G and Beyond)

Three symbiotic trends emerging:

- Deployment of pico- and femto-cells (OoM decrease in cell size)
- Millimeter wave frequencies (OoM increase in bandwidth)
- Massive MIMO (OoM increase in antennas)







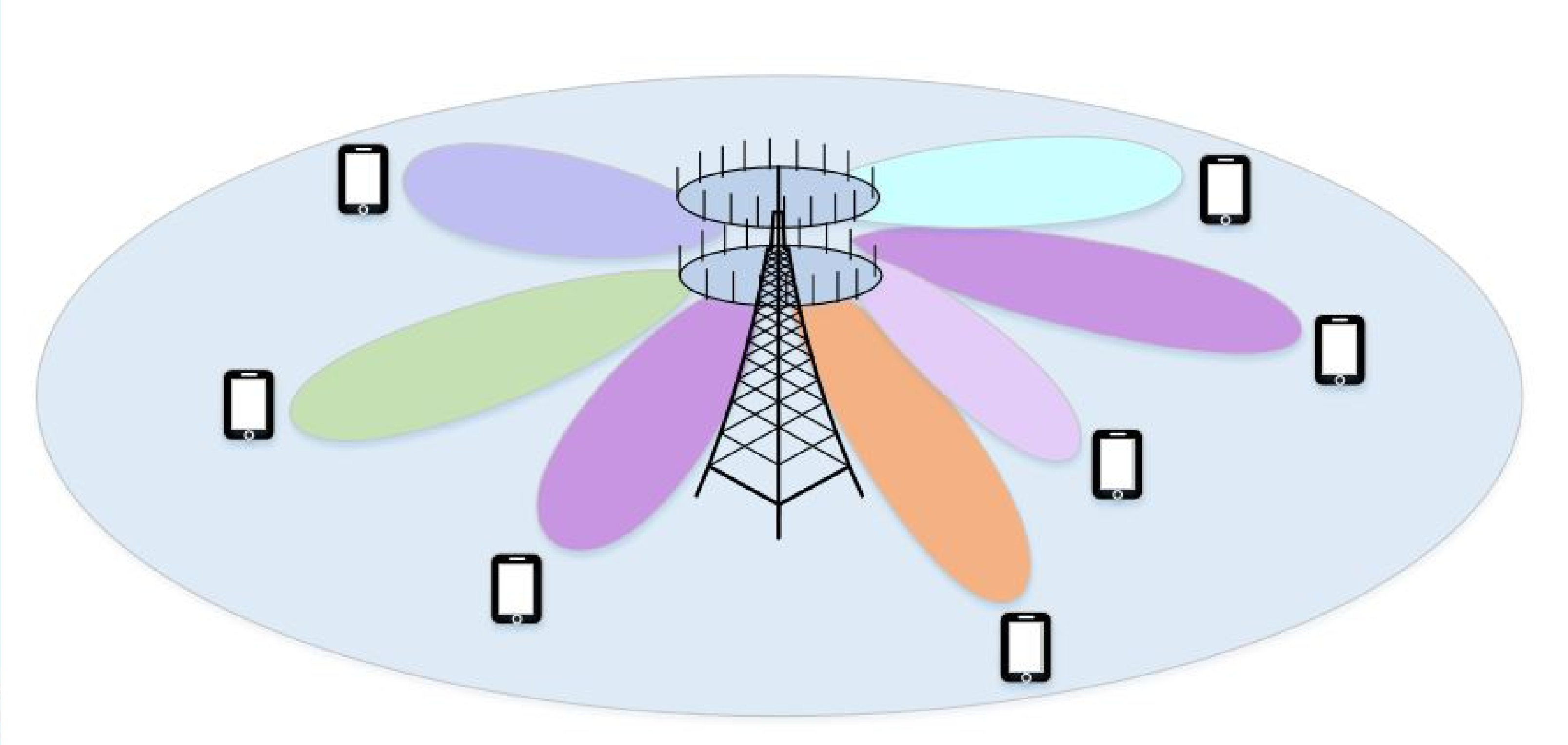
A Symbiotic Relationship

- Millimeter wave frequencies
  - short wavelengths
  - larger propagation losses, shorter range operation
  - little multipath, line-of-sight (LOS) or near-LOS
  - low SNR
  - larger Doppler shifts, more sensitive to mobility

- Massive antenna arrays
  - large array gain
  - size proportional to wavelength
  - narrow, focused beamforming

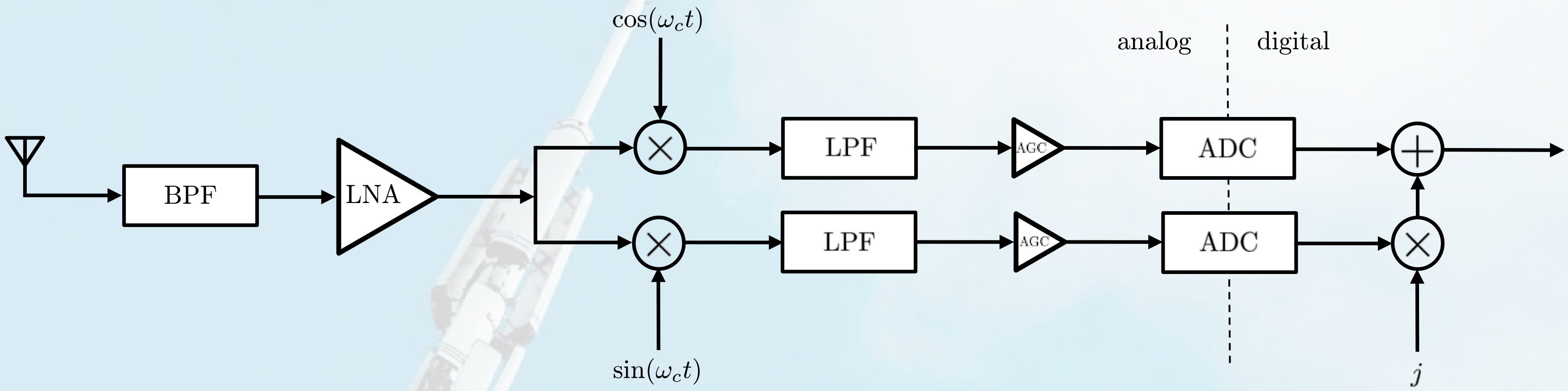
- Small cells
  - short range
  - lower power
  - low mobility
  - interference-limited

Energy Efficiency in Massive MIMO



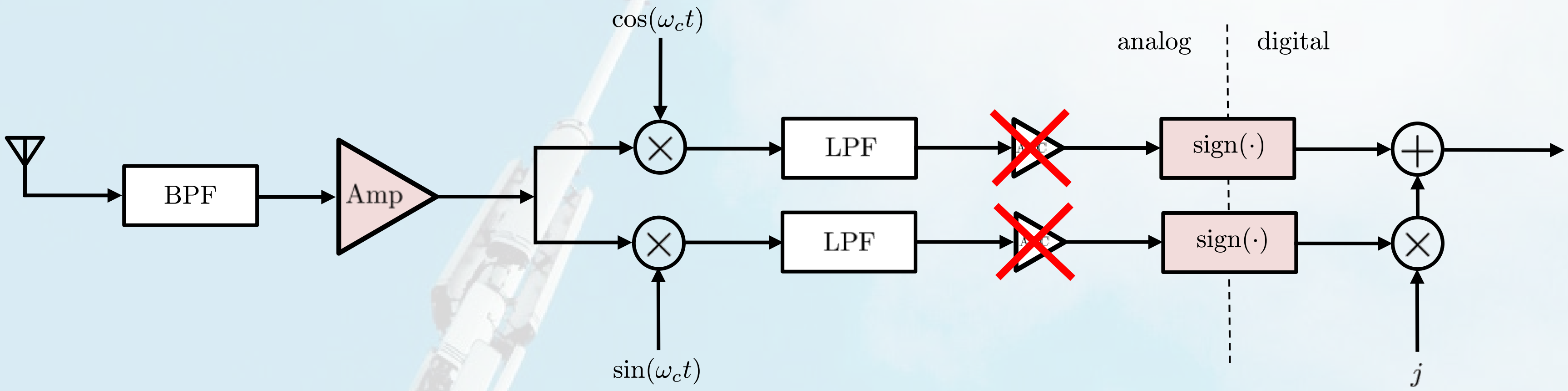
- Energy efficiency and hardware complexity are important issues for massive MISO/MIMO systems
- Use low fidelity hardware (e.g., one-bit ADCs/DACs) to minimize power consumption, low PAPR waveforms to (1) lower OOB interference and spectral regrowth and (2) allow PAs to operate with no back-off

Standard Receiver Implementation



- Full precision ADC requires linear, low-noise amplifiers and AGC
- ADC power consumption grows exponentially with resolution
- A commercial TI 1 Gs/s 12-bit ADC requires as much as 2-4W
- 1 GHz bandwidth, 128 antennas, 12 bit ADC $\longrightarrow$ over 6 Tb/sec into DSP
- Not practical for ideal massive MIMO

A One-Bit Receiver



- One-bit ADC $\Rightarrow$ simple RF, no AGC or high cost LNA
- Operates at a fraction of the power (mW)
- Compensate for quantization error with signal processing, more antennas

Single Antenna Theoretical Analysis

AWGN Channel Capacity

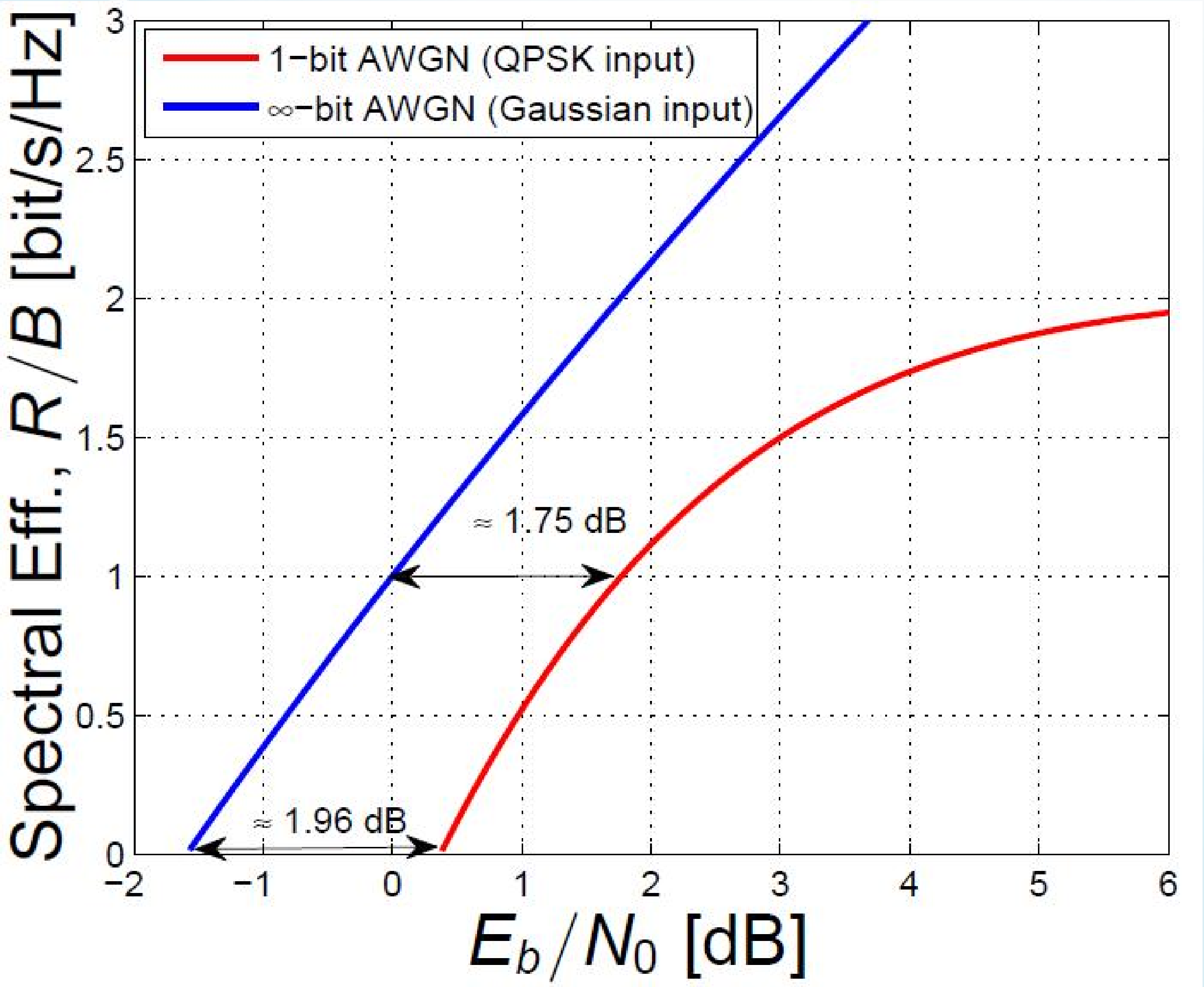
$$B \log_2(1 + \mathrm{SNR})$$

1-Bit AWGN Capacity

$$2B\left(1 - H_b(\Phi(\sqrt{\mathrm{SNR}}))\right)$$

loss in power efficiency < 2dB when SE < 1.4 bpcu

trade-off between power and energy efficiency is less apparent for 1-bit systems



Nonlinear Analysis: The Bussgang Decomposition

Consider a Gaussian signal that passes through a non-linear operator:

$$\mathbf{r} = \mathcal{Q}(\mathbf{x})$$

Bussgang (Bell Labs, 1952) showed that a statistically "equivalent" (up to second order) linear model exists for the non-linearity

$$\mathbf{r} = \mathcal{Q}(\mathbf{x}) = \mathbf{A}\mathbf{x} + \mathbf{q}$$

that results in the error $\mathbf{q}$ (here the quantization noise) and the signal $\mathbf{x}$ being uncorrelated, namely

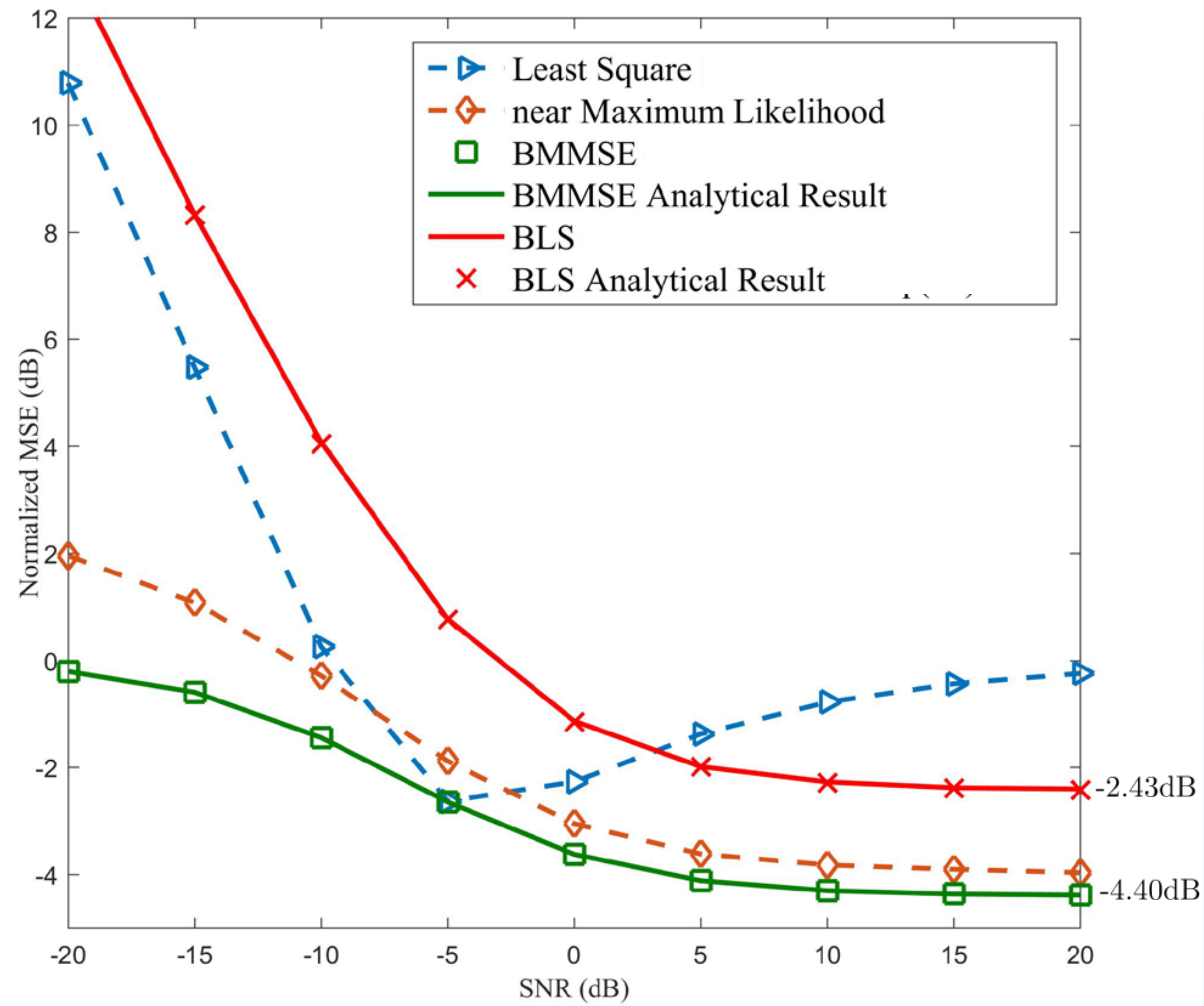
$$\mathbf{A} = \mathcal{E}\{\mathbf{x}\mathbf{r}^T\}\mathcal{E}\{\mathbf{x}\mathbf{x}^T\}^{-1}$$

Linear model that best describes the quantization:

$$\mathbf{A} = \arg\min_{\mathbf{A}} \|\mathbf{r} - \mathbf{A}\mathbf{x}\|^2$$

Channel Estimation using Linear Model

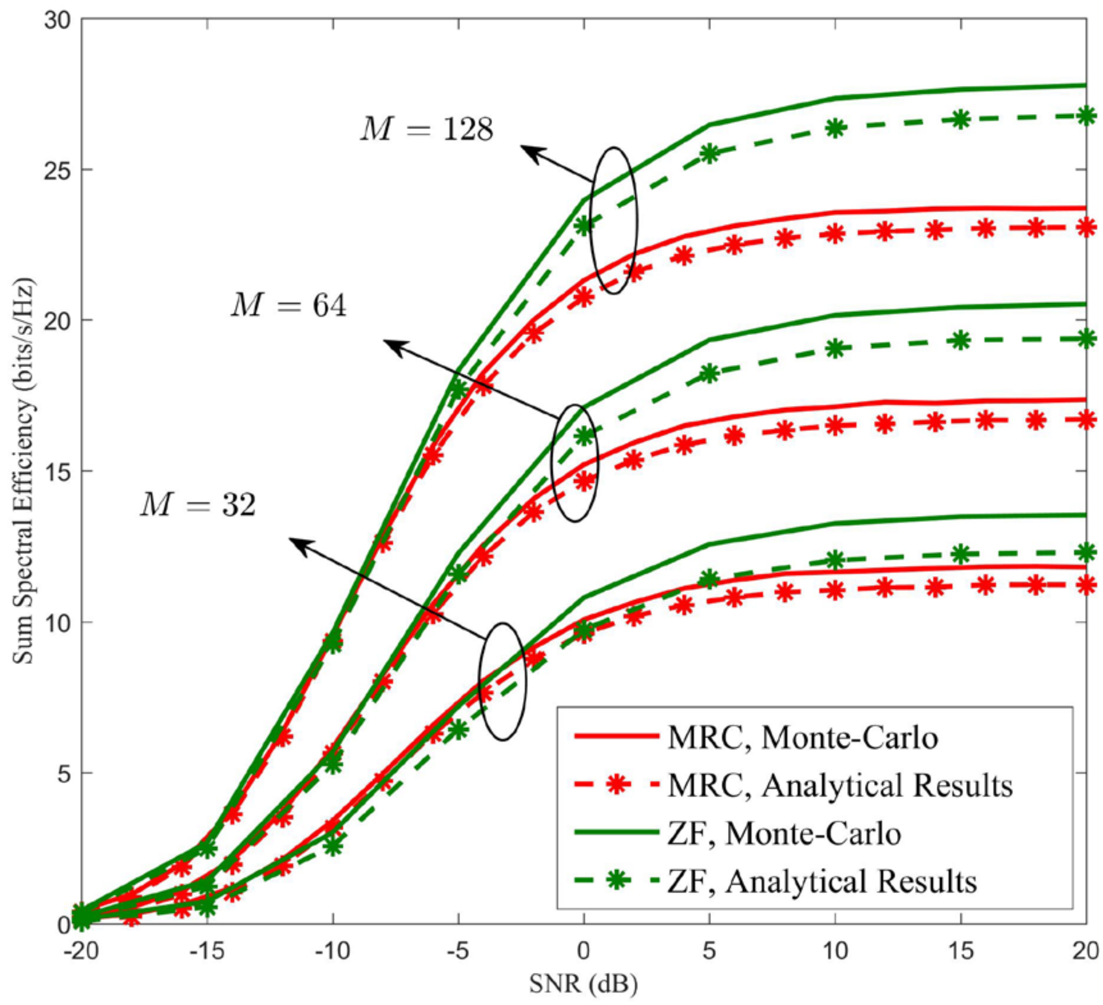
- $M = 128$
- $K = 8$
- $\tau = K$
- Rayleigh fading



Sum Spectral Efficiency

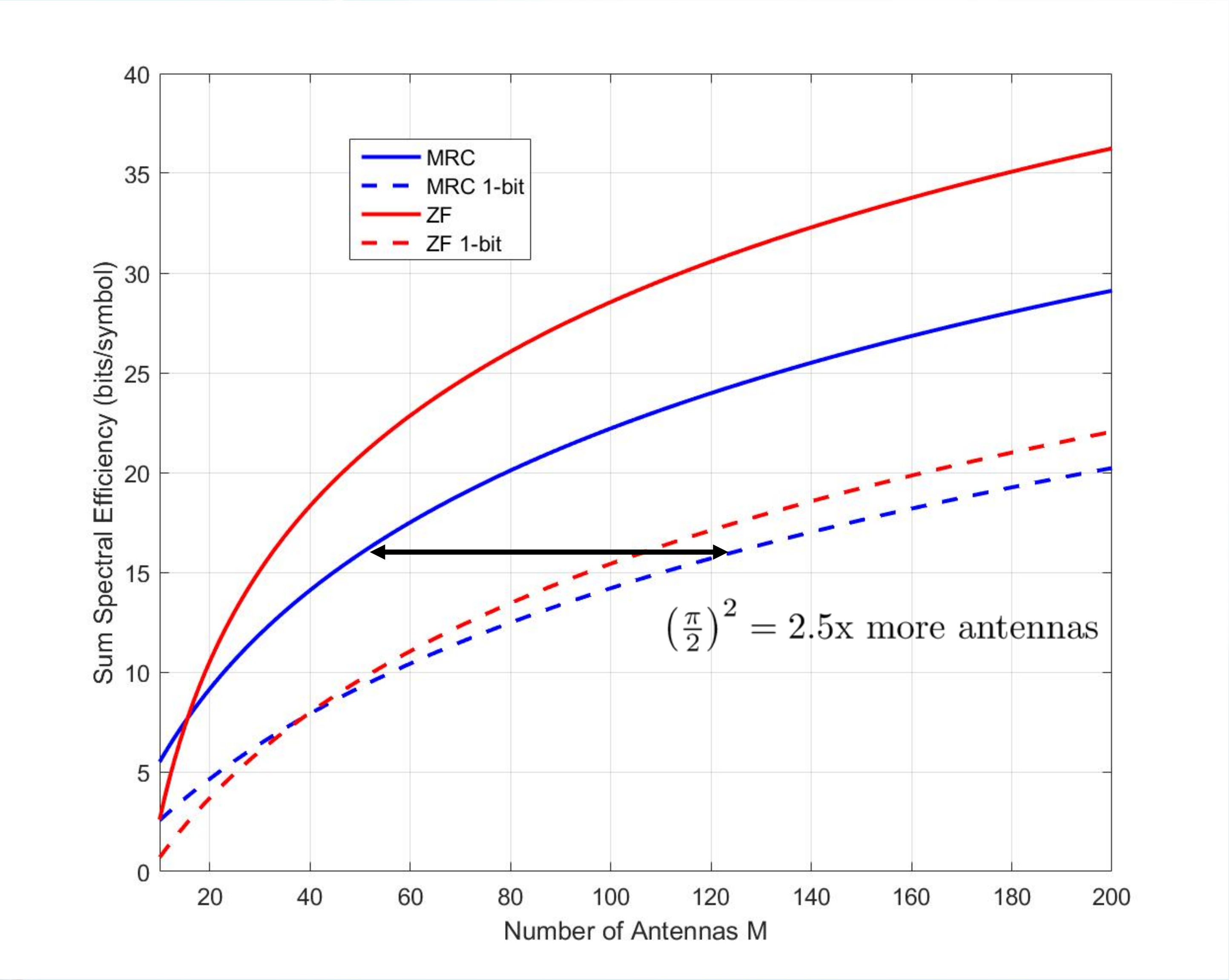
- $K = 8$ users
- equal power
- coherence interval $T = 200$
- sum spec. eff.

$$\mathcal{S} = \frac{T-K}{T} K R$$



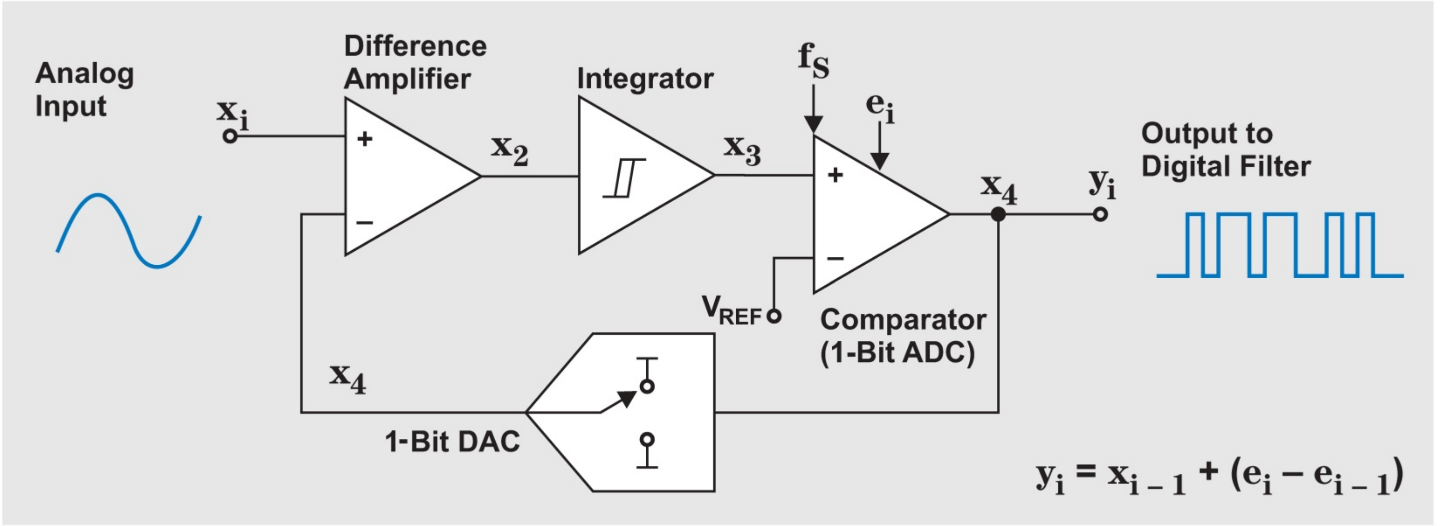
Sum Spectral Efficiency vs. # of Antennas

- flat Rayleigh fading
- -5dB SNR
- $K = 8$ users
- $T = 200$ symbols



Alternative ADC Architecture: Oversampling with ΣΔ ADCs

- Oversampling makes desired signal temporally correlated
- Exploit temporal correlation via feedback, quantization of the error signal
- Requires simple additional analog circuitry

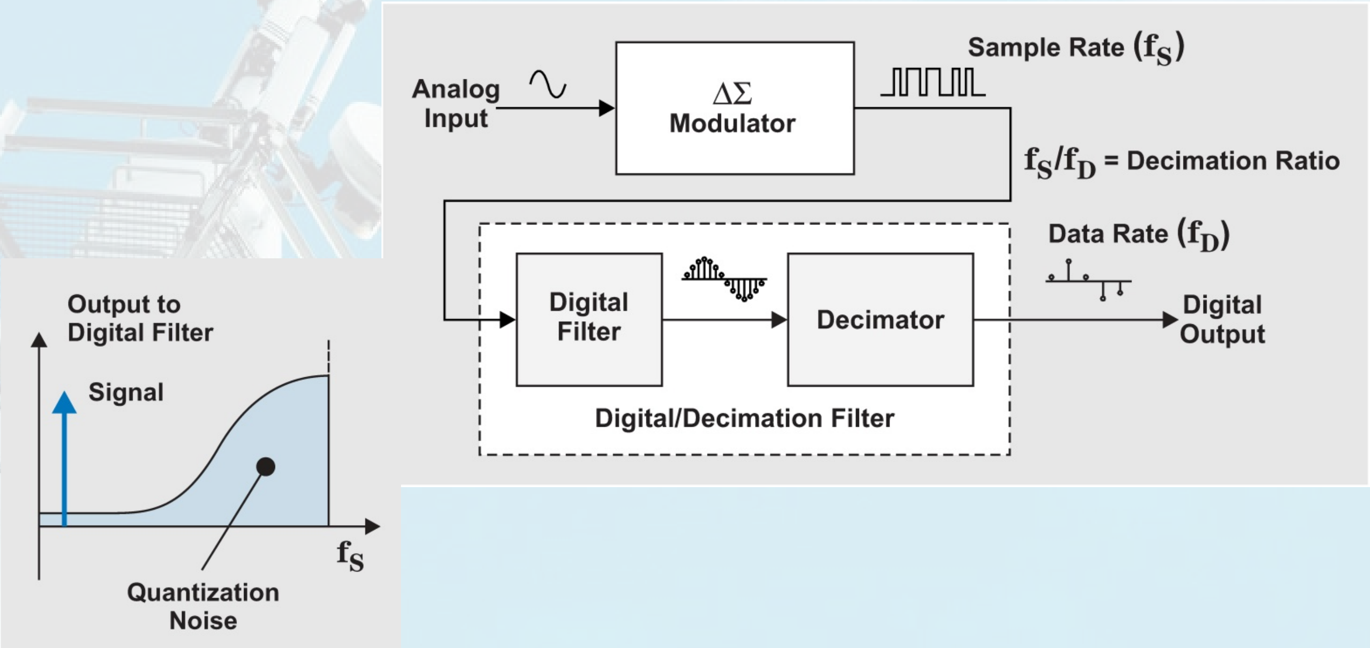


*From Texas Instruments *Analog Applications Journal*

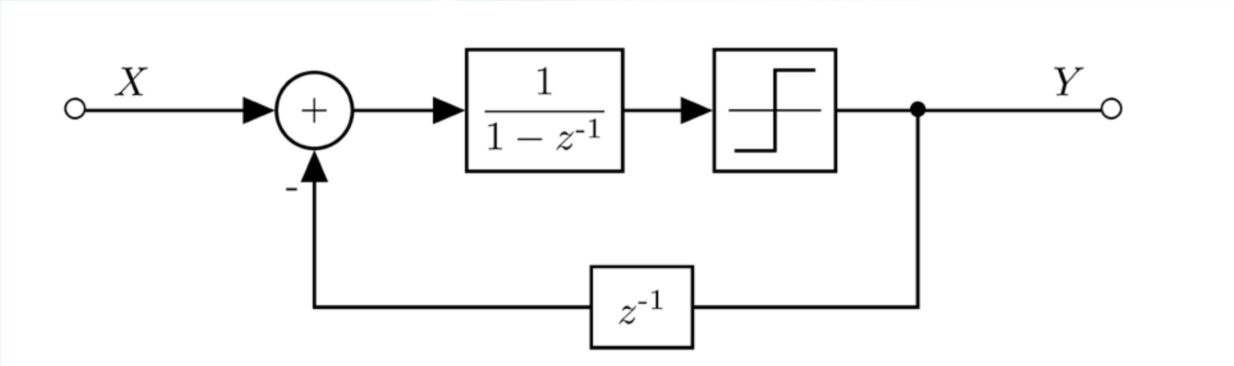
Oversampling with ΣΔ ADCs

- Desired signal pushed to lower frequencies due to oversampling, quantization noise pushed to higher frequencies (noise shaping)

- post-processing low-pass filter and decimation used to recover desired samples



ΣΔ ADC Discrete-Time Equivalent Model

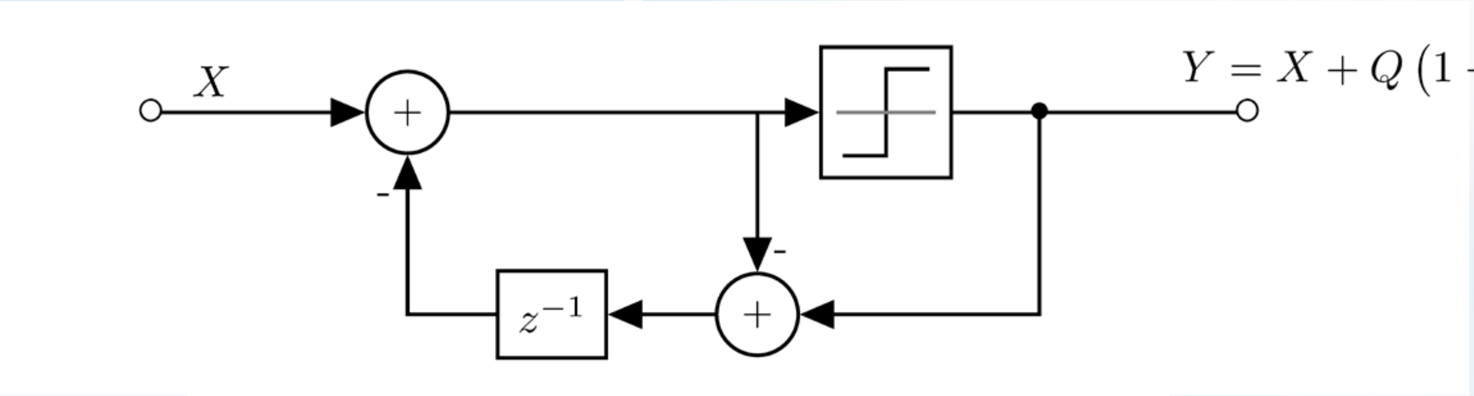


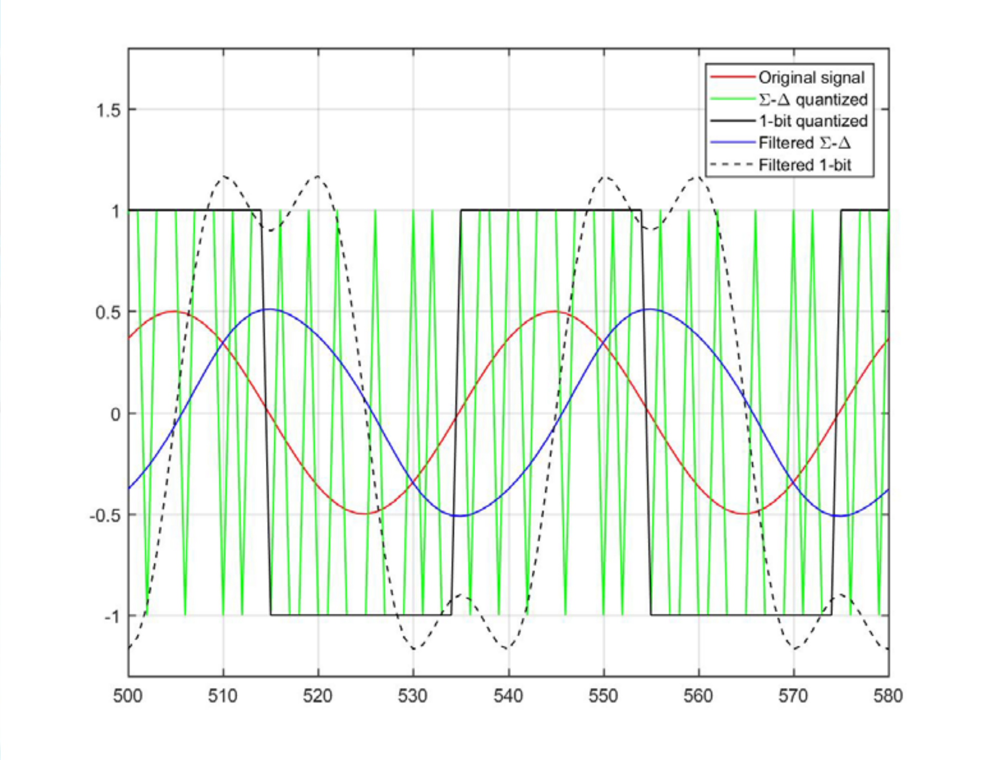
model quantizer as additive noise term Q:

$$Y = \left(-Yz^{-1} + X\right)\frac{1}{1-z^{-1}} + Q$$

$$= X + Q\left(1 - z^{-1}\right)$$

ΣΔ ADC Discrete-Time Equivalent Model, pt. 2





input $X = \sin(\pi n/20)$

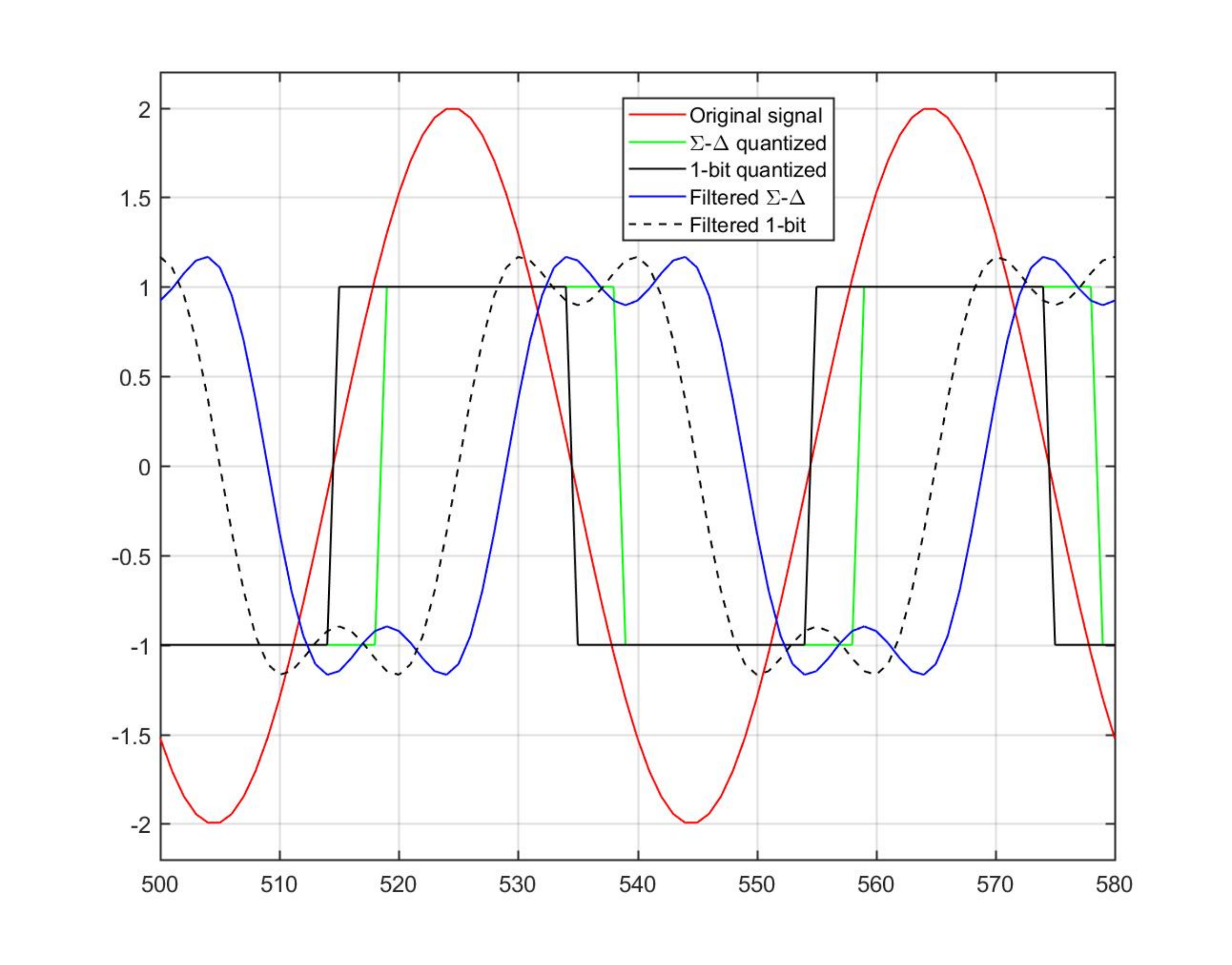
$\Sigma - \Delta$ output

1-bit quantized output

$\Sigma - \Delta + \text{LPF}, \omega_c = \frac{\pi}{6}$

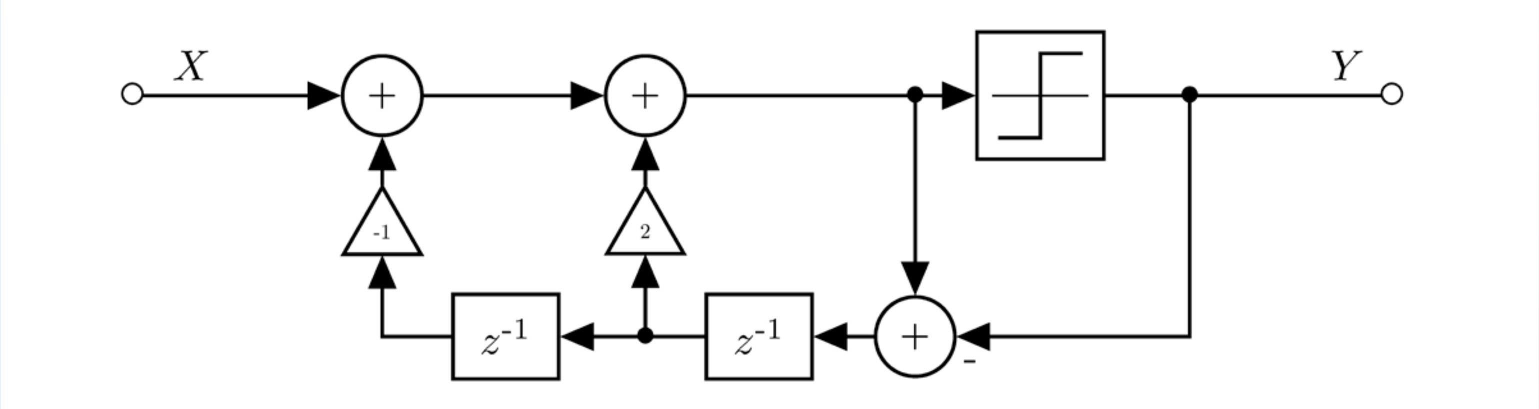
1-bit + LPF

Unlike 1-Bit ADC, ΣΔ ADCs Need Gain Control

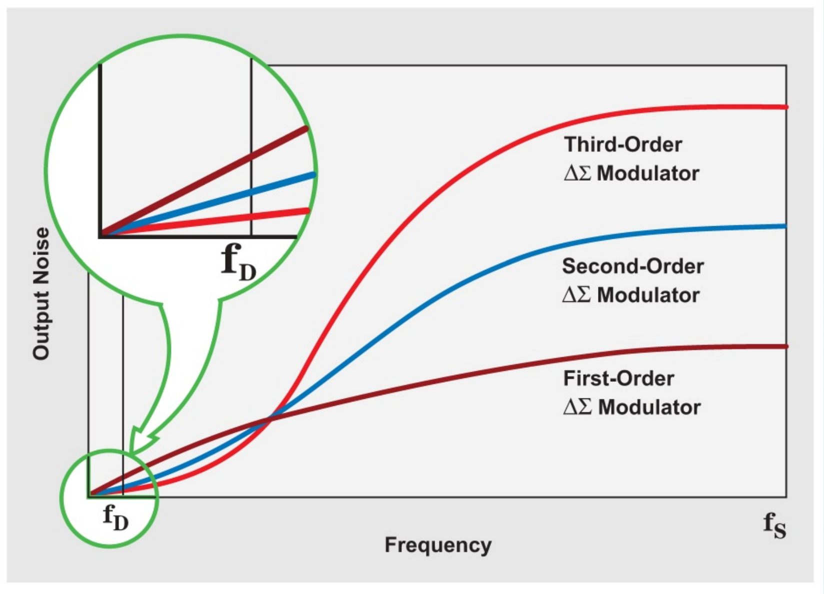


no advantage to ΣΔ here despite oversampling, over-driven ADC

2nd Order ΣΔ ADC

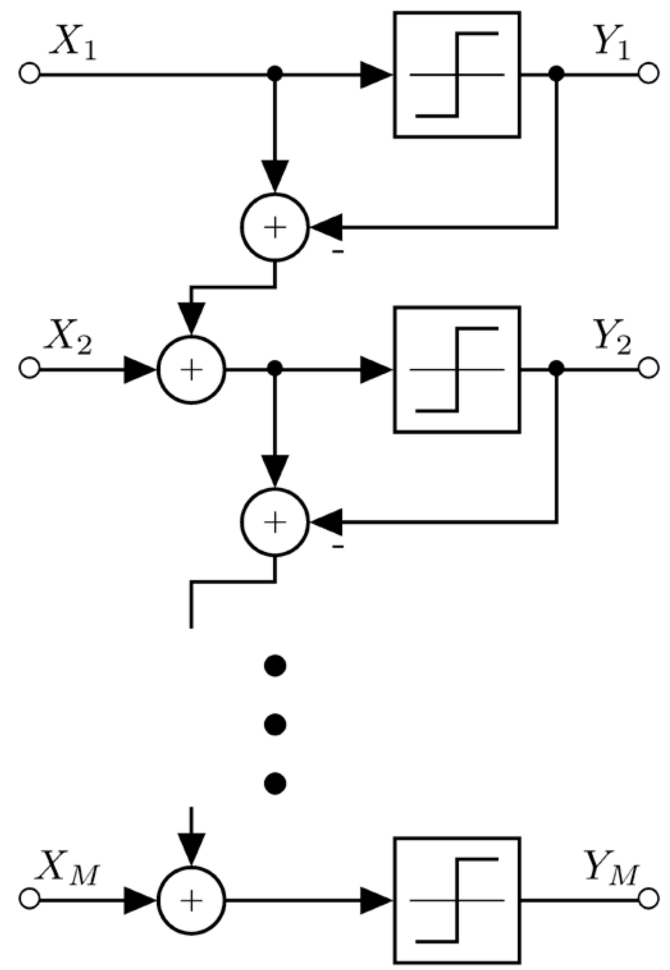


provides further shaping of the quantization noise: $$Y = X + Q\left(1 - z^{-1}\right)^2$$



Spatial ΣΔ Quantization

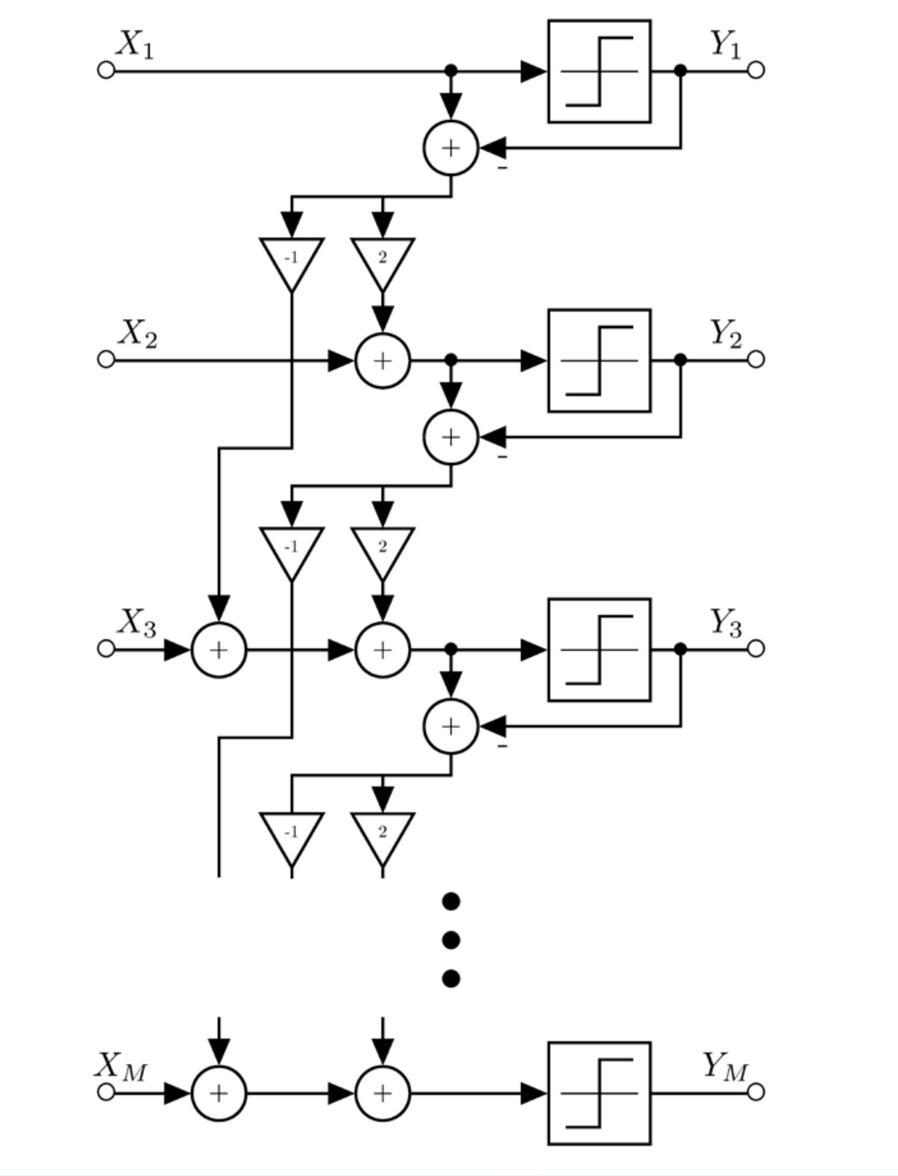
Instead of delayed feedback in time, feedback to adjacent antenna:



Spatial ΣΔ Quantization, cont.

- Oversampling occurs in space rather than time
- For a uniform linear array, this means either antenna spacing less than $\lambda/2$, or sources at low spatial frequencies (nearer to broadside), or both
- Quantization noise is pushed to higher spatial frequencies, so lowpass spatial filtering (beamforming) is needed to reduce impact of quantization
- Second- or higher-order spatial $\Sigma\Delta$ quantization also possible

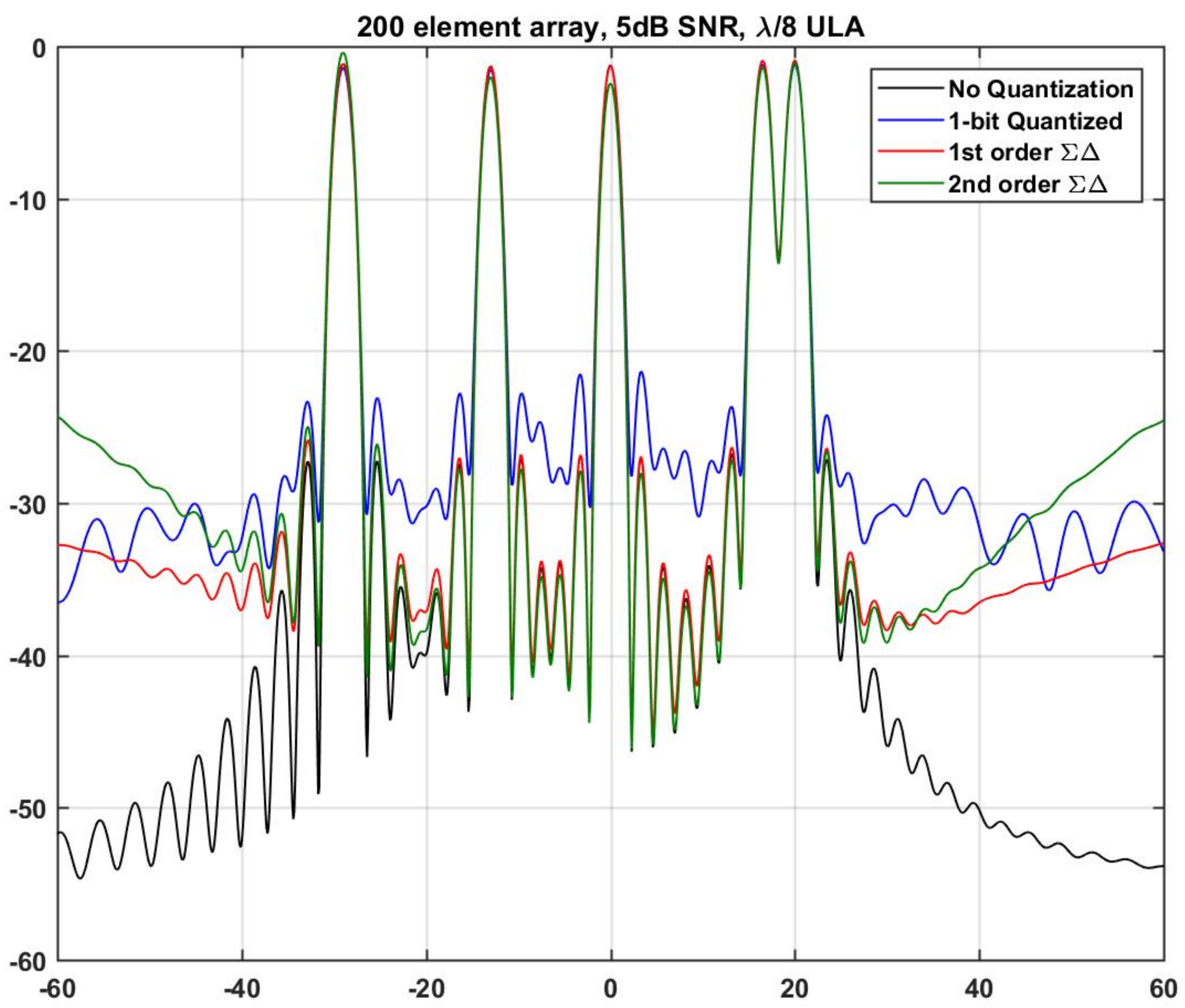
2nd Order Spatial ΣΔ ADC Architecture



Spatial ΣΔ Quantization, Related Prior Work

- C. P. Yeang, G. W. Wornell and L. Zheng, "Oversampling transmit and receive antenna arrays," in *Proc. IEEE ICASSP*, 2010.
- V. Venkateswaran and A. J. van der Veen, "Multichannel ΣΔ ADCs With Integrated Feedback Beamformers to Cancel Interfering Communication Signals," in *IEEE Trans. on Sig. Proc.*, May 2011.
- J. D. Krieger, C. P. Yeang and G. W. Wornell, "Dense Delta-Sigma Phased Arrays," in *IEEE Trans. Ant. & Prop.*, April 2013.
- R. M. Corey and A. C. Singer, "Spatial Sigma-Delta Signal Acquisition for Wideband Beamforming Arrays," in *Proc. 20th International ITG Workshop on Smart Antennas (WSA)*, 2016.
- D. Barac and E. Lindqvist, *Spatial ΣΔ Modulation in a Massive MIMO Cellular System*, MS Thesis, Dept. of Computer Sci. & Eng., Chalmers University of Technology & University of Gothenburg, 2016.
- D. S. Palguna, D. J. Love, T. A. Thomas and A. Ghosh, "Millimeter Wave Receiver Design Using Low Precision Quantization and Parallel ΔΣ Architecture," in *IEEE Trans. Wireless Comm.*, Oct. 2016.
- A. Nikoofard et al., "Low-complexity N-port ADCs using 2-D ΔΣ noise-shaping for N-element array receivers," in *Proc. 60th IEEE Int'l Midwest Symp. on Circuits and Systems (MWSCAS)*, 2017.
- A. Madanayake, N. Akram, S. Mandal, J. Liang and L. Belostotski, "Improving ADC figure-of-merit in wideband antenna array receivers using multidimensional space-time delta-sigma multiport circuits," In *Proc. 10th Int'l Workshop on Multidimensional (nD) Systems (nDS)*, 2017.

Sample Beampatterns Obtained with Spatial ΣΔ ADCs



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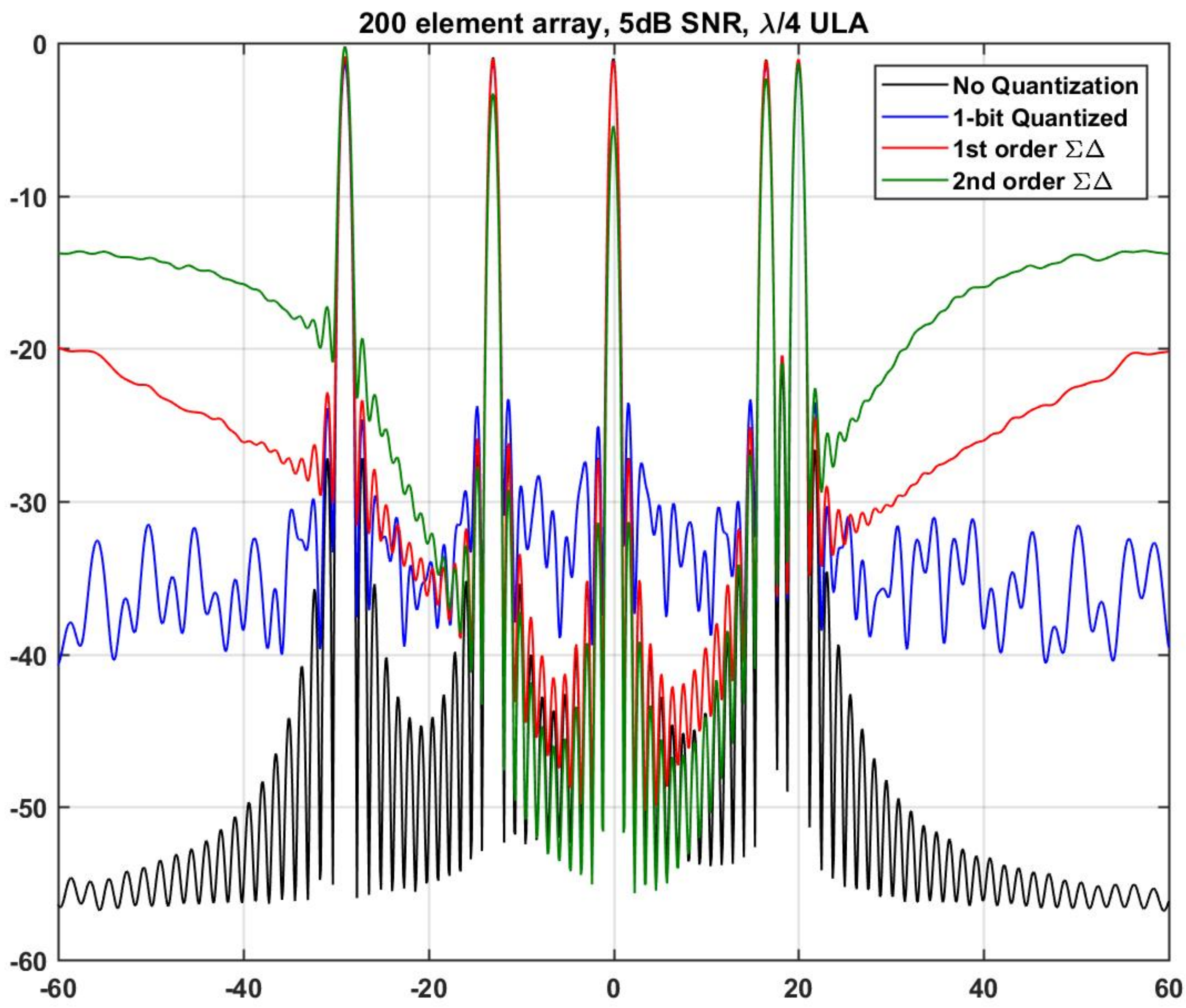

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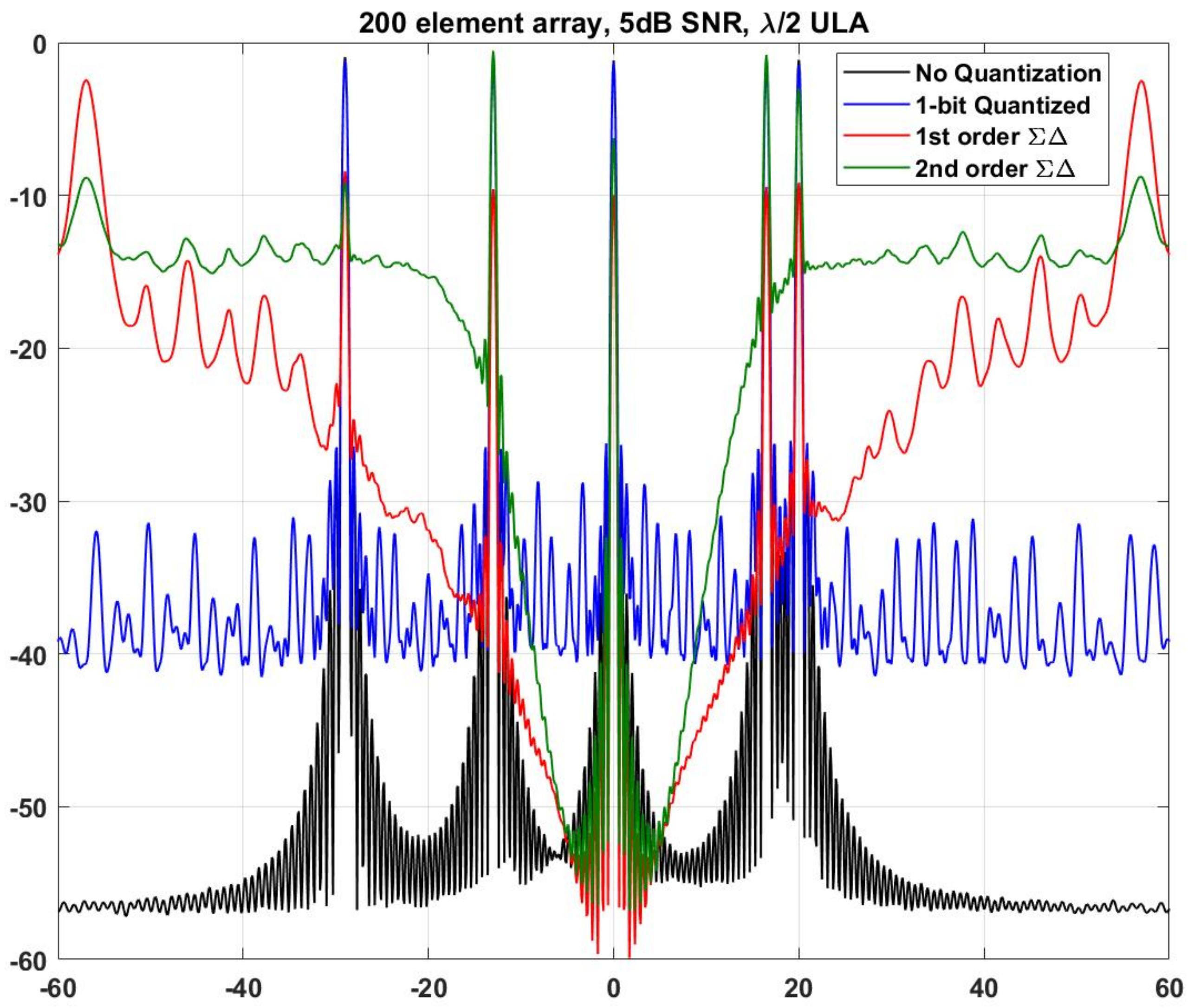
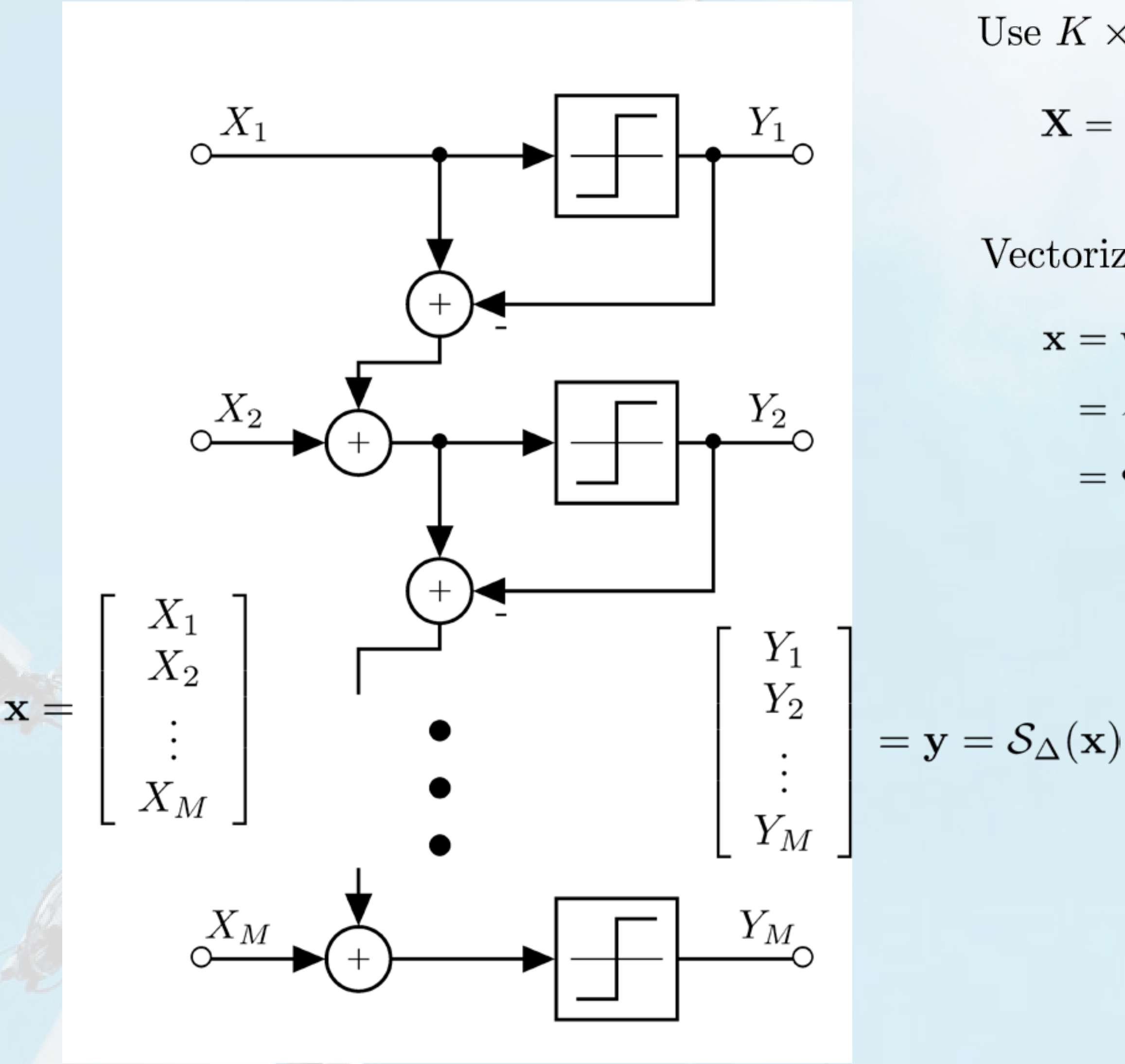


Channel Estimation



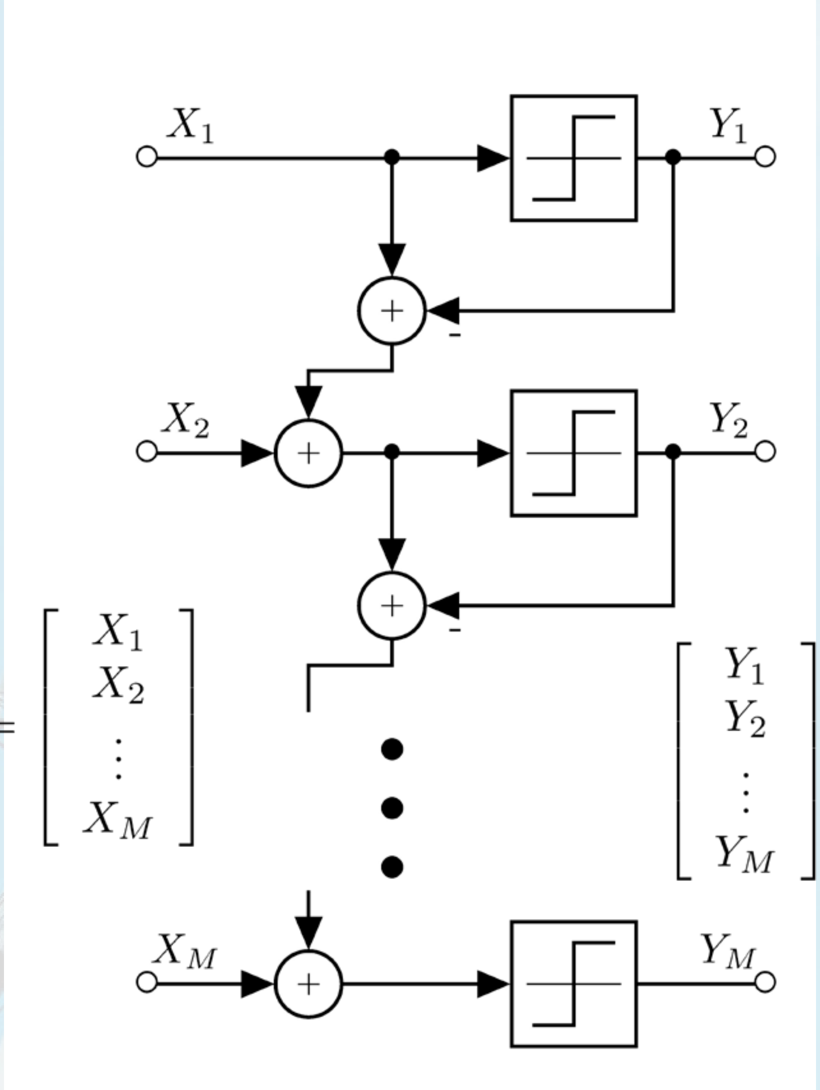
Use $K \times \tau$ uplink training data $\mathbf{\Phi}_t$

$$\mathbf{X} = \sqrt{\rho}\mathbf{H}\mathbf{\Phi}_t + \mathbf{N}$$

Vectorized model

$$\begin{aligned} \mathbf{x} &= \mathrm{vec}(\mathbf{X}) \\ &= \sqrt{\rho}\left(\mathbf{\Phi}_t^T \otimes \mathbf{I}\right)\mathrm{vec}(\mathbf{H}) + \mathrm{vec}(\mathbf{N}) \\ &= \mathbf{\Phi}\mathbf{h} + \mathbf{n} \end{aligned}$$

Channel Estimation



$$\mathbf{x} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_M \end{bmatrix} \qquad \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_M \end{bmatrix} = \mathbf{y} = \mathcal{S}_\Delta(\mathbf{x})$$

Use $K \times \tau$ uplink training data $\mathbf{\Phi}_t$

$$\mathbf{X} = \sqrt{\rho}\mathbf{H}\mathbf{\Phi}_t + \mathbf{N}$$

Vectorized model

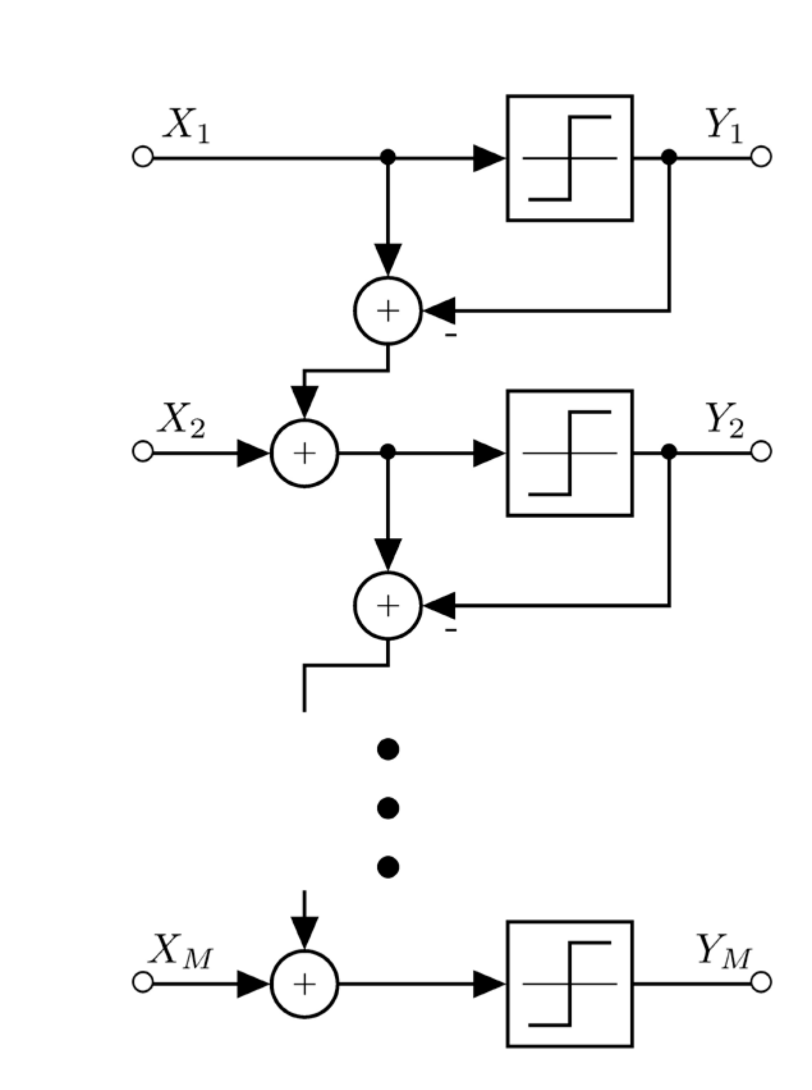
$$\begin{aligned} \mathbf{x} &= \mathrm{vec}(\mathbf{X}) \\ &= \sqrt{\rho}\left(\mathbf{\Phi}_t^T \otimes \mathbf{I}\right)\mathrm{vec}(\mathbf{H}) + \mathrm{vec}(\mathbf{N}) \\ &= \mathbf{\Phi}\mathbf{h} + \mathbf{n} \end{aligned}$$

LPF array output

$$\mathbf{r} = \mathbf{G}\mathbf{y}$$

$M' \times M \; , \; M' < M$

Bussgang Analysis



$$\mathbf{y} = \mathcal{S}_\Delta(\mathbf{x}) = \mathcal{Q}\left(\mathbf{U}\mathbf{x} - \underbrace{(\mathbf{U} - \mathbf{I})}_{\boldsymbol{\Gamma}}\mathbf{y}\right)$$

for first-order $\Sigma\Delta$:

$$\mathbf{U} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & 0 & & 0 \\ 1 & 1 & 1 & 0 & & 0 \\ & & & & \vdots & \\ 1 & 1 & 1 & 1 & \cdots & 1 \end{bmatrix}$$

for second-order $\Sigma\Delta$:

$$\mathbf{U} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 2 & 1 & 0 & 0 & & 0 \\ 3 & 2 & 1 & 0 & & 0 \\ & & & & \vdots & \\ M & M-1 & M-2 & M-3 & \cdots & 1 \end{bmatrix}$$

Bussgang LMMSE Channel Estimate

$$\begin{aligned}\mathbf{y} &= \mathcal{Q}\left(\mathbf{U}\mathbf{x} - \mathbf{\Gamma}\mathbf{y}\right) \\ &= \mathbf{A}\,(\underbrace{\mathbf{U}\mathbf{x} - \mathbf{\Gamma}\mathbf{y}}_{\mathbf{z}}) + \mathbf{q} \\ &= \left(\mathbf{I} + \mathbf{A}\mathbf{\Gamma}\right)^{-1}\mathbf{A}\mathbf{U}\mathbf{x} + \left(\mathbf{I} + \mathbf{A}\mathbf{\Gamma}\right)^{-1}\mathbf{q}\end{aligned}$$

$$\mathbf{A} = \sqrt{\frac{2}{\pi}}\mathrm{diag}\left(\mathbf{C}_z\right)^{-\frac{1}{2}}$$

$$\mathbf{C}_z = \mathbf{U}\mathbf{C}_x\mathbf{U}^H + \mathbf{\Gamma}\mathbf{C}_y\mathbf{\Gamma}^H - \mathbf{U}\mathbf{C}_x\mathbf{U}^H\mathbf{A}^H(\mathbf{I} + \mathbf{\Gamma}^H\mathbf{A}^H)^{-1}\mathbf{\Gamma}^H - \mathbf{\Gamma}(\mathbf{I} + \mathbf{A}\mathbf{\Gamma})^{-1}\mathbf{A}\mathbf{U}\mathbf{C}_x\mathbf{U}^H$$

$$\mathbf{C}_y = \frac{2}{\pi}\left(\arcsin\left(\frac{\pi}{2}\mathbf{A}\mathrm{Re}(\mathbf{C}_z)\mathbf{A}^H\right) + j\arcsin\left(\frac{\pi}{2}\mathbf{A}\mathrm{Im}(\mathbf{C}_z)\mathbf{A}^H\right)\right)$$

1. Calculate $\mathbf{C}_z(1,1)$
2. From $\mathbf{C}_z(1,1)$, find $\mathbf{C}_z(1,2), \mathbf{C}_z(2,1)$
3. Use $\mathbf{C}_z(1,2), \mathbf{C}_z(2,1)$ to determine $\mathbf{C}_z(2,2)$
4. Calculate $\mathbf{C}_z(1,3), \mathbf{C}_z(2,3), \mathbf{C}_z(3,3)$ using previous values
5. etc.

Bussgang LMMSE Channel Estimate

$$\begin{aligned}\mathbf{y} &= \mathcal{Q}\left(\mathbf{U}\mathbf{x} - \mathbf{\Gamma}\mathbf{y}\right)\\ &= \mathbf{A}\underbrace{\left(\mathbf{U}\mathbf{x} - \mathbf{\Gamma}\mathbf{y}\right)}_{\mathbf{z}} + \mathbf{q}\\ &= \left(\mathbf{I} + \mathbf{A}\mathbf{\Gamma}\right)^{-1}\mathbf{A}\mathbf{U}\mathbf{x} + \left(\mathbf{I} + \mathbf{A}\mathbf{\Gamma}\right)^{-1}\mathbf{q}\end{aligned}$$

$$\mathbf{A} = \sqrt{\frac{2}{\pi}}\mathrm{diag}\left(\mathbf{C}_z\right)^{-\frac{1}{2}}$$

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$$\mathbf{C}_y = \frac{2}{\pi}\left(\arcsin\left(\frac{\pi}{2}\mathbf{A}\mathrm{Re}(\mathbf{C}_z)\mathbf{A}^H\right) + j\arcsin\left(\frac{\pi}{2}\mathbf{A}\mathrm{Im}(\mathbf{C}_z)\mathbf{A}^H\right)\right)$$

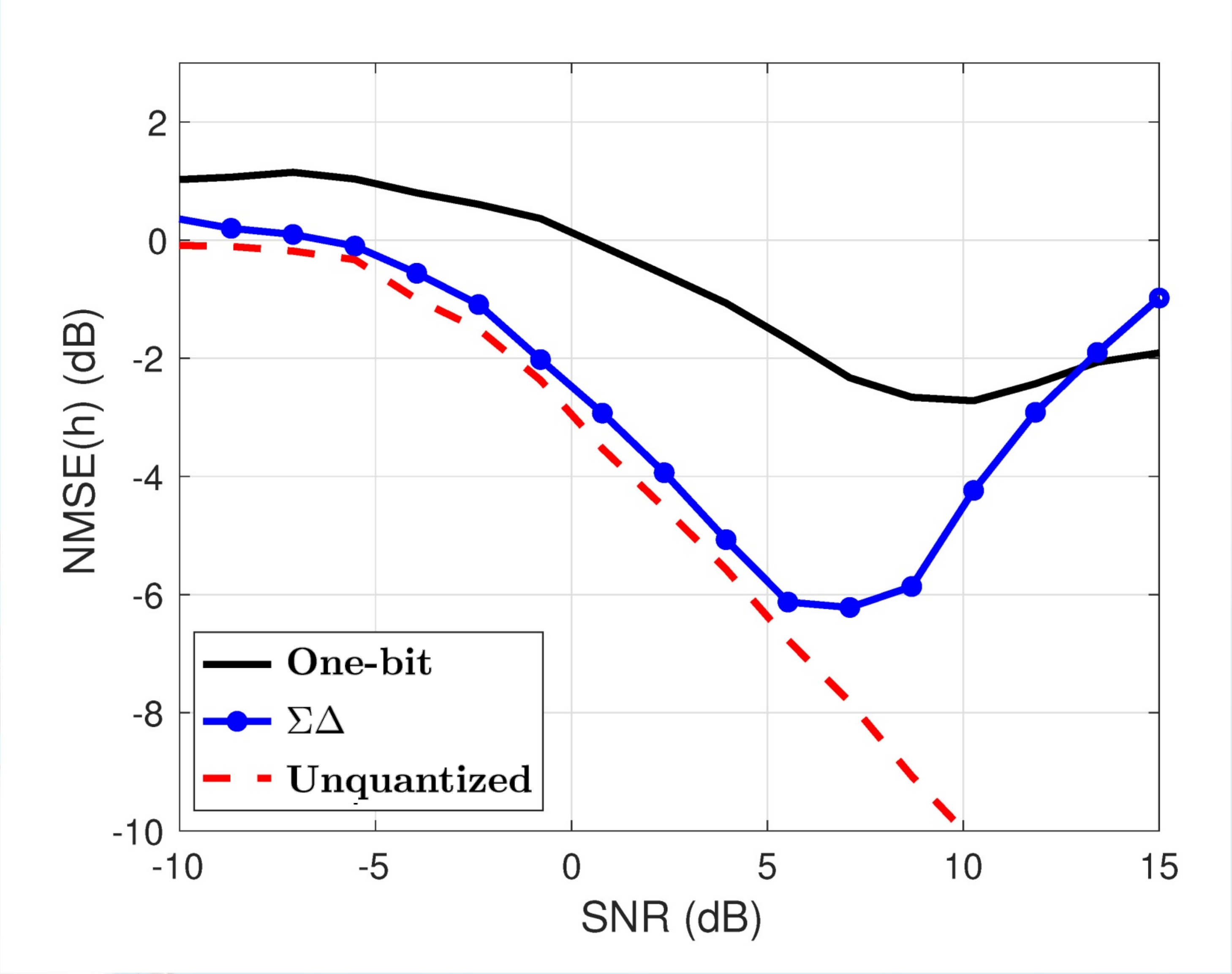
$$\begin{aligned}\hat{\mathbf{h}} &= \mathbf{C}_{hr}\mathbf{C}_{rr}^{-1}\mathbf{r}\\ &= \mathbf{C}_{hy}\mathbf{G}^H(\mathbf{G}\mathbf{C}_y\mathbf{G})^{-1}\mathbf{G}\mathbf{y}\end{aligned}$$

$$\mathbf{C}_{hy} = \mathbf{C}_h\mathbf{\Phi}^H\mathbf{U}^H\mathbf{A}^H(\mathbf{I} + \mathbf{\Gamma}^H\mathbf{A}^H)^{-1}$$

Uplink Simulation with Channel Estimation

- 64 antenna ULA with $\lambda/8$ element spacing
- 8 LoS users with random angles uniformly distributed in $[-30^\circ, 30^c irc]$
- Array output processed with 16-tap low-pass beamformer with cutoff at $\pm 45^\circ$
- Channel estimated using orthogonal pilots of duration 8 samples
- Estimated channels used in ZF receiver to decode subsequent QPSK symbols

Uplink Simulation with Channel Estimation



Conclusions

- Massive MIMO, small cells and mm-wave frequencies provide symbionic benefits for 5G
- Low-resolution (e.g., 1-bit) quantization provides high spectral efficiency and significant energy savings
- One-bit $\Sigma\Delta$ ADC architectures provide gains in situations where users have low spatial frequencies (bigger gains likely for higher-dimensional constellations, e.g., 16-QAM)
- Realistic system simulations show 2-4 bit ADCs yield best energy-spectral efficiency trade-off